[3] Blottner, F. G., "Nonequilibrium laminar flow of ionized air," AIAA Preprint 64-41 (January 1964).

[4] Li, H., "Study of hypersonic contaminated wake by an exact numerical solution," General Electric Co. Rept. MSD-R63SD63 (December 1963).

[5] Pallone, A., Moore, J., and Erdos, J., "Nonequilibrium nonsimilar solutions of the laminar boundary layer equations," AIAA Preprint 64-40 (January 1964).

[6] Levinsky, E. S. and Fernandez, F. L., "Approximate nonequilibrium air ionization in hypersonic flows over sharp cones," AIAA J. 2, 565–568 (1964).

[7] Pallone, A. J., Erdos, J. I., Eckerman, J., and McKay, W., "Hypersonic laminar wakes and transition studies," AIAA Preprint 63-171 (June 1963).

[8] Webb, W. H. and Hromas, L. A., "Turbulent diffusion of a reacting wake," AIAA Preprint 64-42 (January 1964).

[9] Erdos, J. and Pallone, A., "Correlation of numerical solutions of the laminar wake," AIAA J. (submitted for publication).

[10] Hromas, L. A. and Lees, L., "Effect of nose bluntness on the turbulent hypersonic wake," Space Technology Lab. Rept. BSD-TDR-62-354 (October 1962).

Asymmetrical Buckling of Spherical Caps under Uniform Pressure

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Nomenclature

- a = radius of curvature of shell
- E = Young's modulus
- H = height of shell center above plane through the edge
- w = normal displacement of shell midsurface
- N_θ = circumferential average stress
- h = shell thickness
- n = number of circumferential waves in buckling mode
- ν = Poisson's ratio
- λ = geometrical parameter = $2[3(1-\nu^2)]^{1/4}(H/h)^{1/2}$
- q = uniform normal pressure
- q_{cr} = buckling pressure
- q_0 = classical buckling pressure of complete spherical shell = $2\,Eh^2/a^2[3(1-\nu^2)]^{1/2}$
- p = buckling pressure ratio = q_{cr}/q_0

SEVERAL investigators have explored the possibility that accurate buckling loads for shells of revolution under axisymmetric loads can be obtained by computing axisymmetric equilibrium positions using nonlinear finite deflection theory and then checking these equilibrium states for points of bifurcation into asymmetric buckling modes that differ from the prebuckled modes by infinitesimal amounts. The buckling criterion is the same as in classical bifurcation theory, but the assumption of the linearized theory that the prebuckled state of stress can be represented by membrane theory has been discarded. Stein[1a] has used this approach to compute the effect of simply supported edges on the stress distribution and buckling loads of cylinders under axial compression and uniform pressure. Huang,[2] Weinitschke,[1b] and Parmerter and Fung[1c] have studied the asymmetric buckling of clamped shallow spherical shells under uniform pressure.

Table 1 Calculated buckling pressures p

λ	n	This note	Huang[5]
6	2	0.777	0.775
	3	0.814	0.827
	1	0.943	0.919
8	4	0.756	0.766
	3	0.778	0.774
	2	0.886	0.893
12	7	0.789	0.780
	8	0.794	0.790
	6	0.804	0.798
16	10	0.789	0.792
	12	0.803	0.800
	11	0.810	0.790
20	15	0.806	...
	16	0.806	...
	17	0.826	...

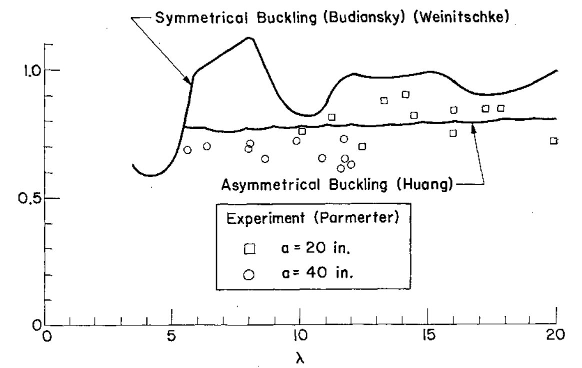

Fig. 1 Stability curves compared with Parmerter's experimental results.

This note presents buckling pressures for spherical caps obtained from a digital computer program[3] that is based on the same type of analysis as the forementioned references but is applicable to general shells of revolution. The program computes buckling stresses due to external pressure, axial load, torsion and axisymmetric temperature gradients, or combinations of these. It can also compute natural frequencies of free vibration of shells under all of these loads except torsion.

This solution is restricted to thin isotropic shells of revolution. The thickness and Young's modulus of the shell can vary continuously in the meridional direction, but not in the circumferential direction. The boundary conditions can be any linear combination of stress resultants and deflections.

The buckling pressures for clamped spherical caps reported by Huang do not agree with those of Weinitschke. Parmerter[4] has obtained results for a limited range of λ that agree with Huang's results. Table 1 lists buckling pressures from the present computer program and corresponding data from Huang's report.[5] Huang's results are from shallow-shell theory, whereas the pressures in this note are from a more general theory with a semiapex angle of the shell of 11.5°; both sets of calculations used $\nu = \frac{1}{3}$. The results of the two theories show good agreement.

Figure 1 shows the envelope of Huang's buckling pressures plotted as a function of λ along with some recent experimental results reported by Parmerter and the curve for axisymmetric snap buckling obtained by Budiansky[6] and by Weinitschke.[7]

Parmerter's experimental pressures are for near perfect shells and some are actually higher than the theoretical curve for bifurcation buckling, although they lie below the theoretical curve for snap buckling. A tentative explanation could be that the asymmetric post-buckling equilibrium states are stable at pressures near the bifurcation point. For large values of λ, the axisymmetric hoop stress and the asymmetric normal deflection mode shape attain their maximum values near the clamped boundary† (see Fig. 2). It is possible that the clamped spherical cap can wrinkle near the edge in a manner analogous to that of rectangular plates in compression. A confirmation of this assumption would require actual asymmetric solutions of the finite deflection equations for spherical

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† Huang[5] derives an asymptotic solution for the boundary-layer effect on the mode shape.

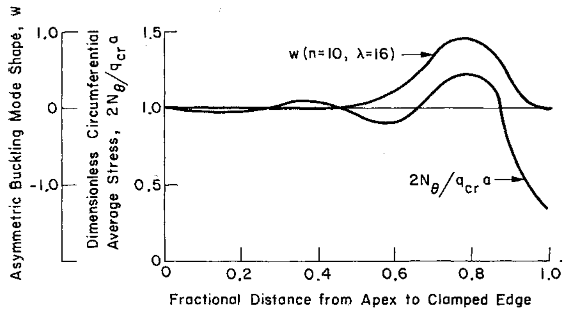

Fig. 2 Boundary-layer effect in circumferential average stress and asymmetric buckling mode.

caps rather than merely showing their existence at bifurcation pressures.

References

[1] *Collected Papers on Instability of Shell Structures,* NASA TN D-1510 (1962): a) Stein, M., "The effect on the buckling of perfect cylinders of prebuckling deformations," pp. 217–227; b) Weinitschke, H. J., "Asymmetric buckling of clamped shallow spherical shells," pp. 481–490; c) Parmerter, R. R. and Fung, Y. C., "On the influence of nonsymmetrical modes on the buckling of shallow spherical shells under uniform pressure," pp. 491–499.

[2] Huang, N. C., "Unsymmetrical buckling of thin shallow spherical shells," AIAA J. **1**, 945–946 (1963).

[3] Thurston, G. A., "Asymmetric buckling of shells of revolution with axisymmetric initial imperfections," General Electric Co. Rept. R62FPD426 (1963).

[4] Parmerter, R. R., "The buckling of clamped shallow spherical shells under uniform pressure," Air Force Office of Scientific Research Rept. 5362 (1963).

[5] Huang, N. C., "Unsymmetrical buckling of thin shallow spherical shells," Harvard Univ. TR 15 (1963).

[6] Budiansky, B., "Buckling of clamped shallow spherical shells," *Proceedings of IUTAM Symposium on the Theory of Thin Elastic Shells* (North Holland Publishing Co., Amsterdam, 1960), pp. 64–94.

[7] Weinitschke, H. J., "On the stability problem of shallow spherical shells, J. Math. Phys. **38**, 209–231 (1960).

Equivalence of Nonequilibrium Flows behind Normal and Oblique Shock Waves

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DISSOCIATING flow behind a plane oblique shock wave was investigated by Epstein[1] and has recently been elaborated by Hsu and Anderson[2] using a different approach proposed by Sedney.[3] In both cases numerical integrations are required for each shock-wave angle and each freestream Mach number; the numerical computations in Ref. 2 are comparatively involved. Thus far, an important, fundamental aspect of nonequilibrium flow seems to have been overlooked, namely, that just as in the classical case of equilibrium flow, nonequilibrium flow behind an oblique shock wave with curved streamlines and apparently very nonuniform flow properties is *equivalent* to a nonequilibrium flow behind a normal shock wave. This note serves to point out this equivalence.

To see the equivalence physically, one notes that all flow properties are constant along lines parallel to an oblique shock wave[1] and that the velocity component v parallel to the oblique shock wave remains constant throughout the flow field because of the absence of pressure gradient in the direction parallel to the shock wave. Hence, to an observer moving with the speed v along the shock front (Fig. 1), the entire flow field is identical to the flow field in the case of a normal shock wave. To verify this equivalence mathematically, one merely writes the fundamental differential equations in rectangular coordinates with the y axis parallel to the oblique shock wave, keeping in mind that all flow variables are independent of y. The resulting equations are identical to those for normal shock waves. Incidentally, Epstein would have reached the same conclusion had he adopted the shock-oriented coordinates.

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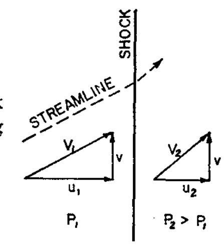
Fig. 1 Equivalence of oblique shock to normal shock to observer moving with velocity v.



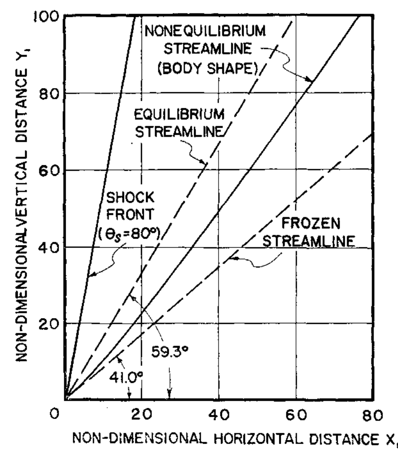


Fig. 2 Streamlines behind an oblique shock wave (θ_s = 80°).

Recognizing this equivalence, one can easily construct solutions for oblique shock waves, given a normal-shock solution. Such plane oblique shock waves with nonequilibrium flow behind them may be considered as being supported by a properly shaped cusped body,[2] or they may be considered as asymptotes to shock waves generated by some two-dimensional bodies, e.g., the wedge.[3] The streamlines (or the shape of the cusped body) can be found through a simple integration; they are necessarily curved since the tangential velocity component remains constant while the normal velocity component varies with the distance normal to the shock wave. A typical streamline constructed by the isocline method is presented in Fig. 2, based on a normal shock solution by Freeman[4] (curve C of Fig. 2a in Ref. 4).

It ought to be noted that this equivalence applies to all nonequilibrium flows, be they dissociating, vibrationally relaxing, or chemically reacting.

References

[1] Epstein, M., "Dissociation relaxation behind a plane, oblique shock wave," J. Aerospace Sci. **28**, 664–665 (1961).

[2] Hsu, C. T. and Anderson, J. E., "Nonequilibrium dissociating flow over a cusped body," AIAA J. **1**, 1784–1789 (1963).

[3] Sedney, R., "Some aspects of nonequilibrium flows," J. Aerospace Sci. **28**, 189–196, 208 (1961).

[4] Freeman, N. C., "Nonequilibrium flow of an ideal dissociating gas," J. Fluid Mech. **4**, 407–425 (1958).